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IIASA Research Memorandum
November 1975

RM-75-57

SPATIAL MIGRATION EXPECTANCIES

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November 1975

Preface

A central concern of the Human Settlements and Services research group at I.I.A.S.A. has been the analysis of the dynamics of multiregional population growth and distribution. Recently this activity has stimulated a concerted effort to extend and expand the applicability of mathematical demographic models in the study of such dynamics. This paper, the fourth of a series addressing the general topic of spatial population dynamics, considers a fundamental problem in migration analysis, namely, the definition of an appropriate index of geographical mobility.

November 1975

Papers in the Spatial Population Dynamics Series

1. Andrei Rogers and Frans Willekens, "Spatial Population Dynamics," RR-75-24, July, 1975, forthcoming in Papers, Regional Science Association, Vol. 36, 1976.

2. Andrei Rogers and Jacques Ledent, "Multiregional Population Projection," IIASA internal working paper, forthcoming in Proceedings, 7th I.F.I.P. Conference, 1976.

3. Andrei Rogers and Jacques Ledent, "Increment-Decrement Life Tables: A Comment," IIASA internal working paper, forthcoming in Demography, 1976.

4. Andrei Rogers, "Spatial Migration Expectancies," RM-75-57, November 1975.

Spatial Migration Expectancies

Andrei Rogers

Abstract

The notion of expectancy is a fundamental concept in demographic analysis and appears frequently in mortality, fertility and migration studies. Such expectancies, however, have not been given a spatial dimension and therefore cannot be differentiated according to places of birth and places of residence. This paper introduces a spatial dimension into the definition of migration expectancies and illustrates their calculation and interpretation with a numerical example.

1. Spatial Expectancies

The notion of expectancy is a fundamental concept in population discourse. Demographers often refer to life expectancies, for example, when speaking about mortality, and to reproduction expectancies when discussing fertility. They have calculated for instance that 73 is the average number of years a female could expect to live under the mortality schedule of the U.S. in 1958, and 1.71 is the average number of baby girls she could expect to bear during her lifetime under the then prevailing fertility schedule. The former measure is known as the expectation of life at birth, e(0); the latter index is called the net reproduction rate, NRR.[1]

Expectancies also have been used in migration studies (Wilber, 1963; Long, 1973). However, their definition has

[1] A related index is the gross reproduction rate, GRR. This measure totally ignores the effects of mortality on reproduction and may be viewed as the net reproduction rate that would arise among a cohort if all of them survived to the end of their childbearing ages. For this reason, the GRR of a population is, of course, always larger than the corresponding NRR.

been nonspatial inasmuch as they view migration as an event in a national population rather than as a flow between regional populations. The study of spatial population dynamics can be considerably enriched by explicitly identifying the locations of events and flows. Such an identification permits one to define spatial expectancies such as the expectation of life at birth or the net reproduction rate of individuals born in region i (respectively, $_ie(0)$ and $_iNRR$, say), and the expected allocation of this lifetime or rate among the various constituent regions of a multiregional population system ($_ie_j(0)$ and $_iNRR_j$, respectively, $j = 1,2,\ldots,m$). For example, it has been estimated (Rogers, 1975) that the expectation of life at birth of a California-born woman exposed to the 1958 U.S. schedules of mortality and migration would be 73.86 years, out of which 24.90 years would be lived outside of California. The net reproduction rate of such a woman, on 1958 fertility rates, would be 1.69, with 0.50 of that total being born outside of California.[2]

Expressing these expectancies as fractions we may define, first, the spatial migration level of California women migrating to the rest of the United States (region j) as

$$_i\theta_j = {}_ie_j(0)/{}_ie(0) = 24.90/73.86 = 0.34 \quad ,$$

and, second, the corresponding spatial net reproduction allocation

$$_i\rho_j = {}_iNRR_j/{}_iNRR = 0.50/1.69 = 0.29 \quad .$$

For expositional simplicity, we have restricted our numerical illustration to a two-region division of the U.S. population. The same concepts can of course be extended to any finite

[2] In calculating these measures it was assumed that the U.S. population was a closed system and, hence, that the region "outside of California" was in fact the rest of the United States.

number of regions. In Table 1, for example, we present the spatial migration expectancies and levels of the female 1968 population of the U.S. disaggregated into four regions, and in Table 2 we list the associated net reproduction rates and allocations. In both tables, high levels of internal migration are associated with high values of ${}_i\theta_j$ and ${}_i\rho_j$ $(i \neq j)$, and conversely.

2. Counts and Duration Times as Expectancies

The first demographer to apply the notion of expectancy to migration analysis carefully distinguished between the two different classes of events which could be studied with the aid of expectancies:

> "A sharp distinction must be drawn between two kinds of events that may be handled in an expectancy table. First, there are events which can occur but once and are non-reversible. The life table illustrates expectancies for this kind of event. A person dies only once and we can calculate the probability of a person dying. Secondly, there are events which can occur several times to an individual and therefore may be reversible and recurrent. Migration, morbidity, marriage, and unemployment are types of events which can and do occur more than once to a person." (Wilber, 1963, p. 445.)

Since the first kind of event occurs only once, it obviously makes no sense to count the number of such events over a person's lifetime. One must, therefore, resort to a duration measure, for example, expected lifetime. In the case of events that can occur several times during a person's lifetime, however, it clearly makes very good sense to count the number of such occurrences. This suggests an enumeration measure, for example, expected number of births.

Adopting the second perspective, Wilber developed a set of migration expectancies describing the average number of moves experienced by an individual during his remaining lifetime. For example, on the basis of 1958 data he concluded:

> "A person who survived to age one in 1958 had an expectancy of 12.99 moves to a different house during his remaining lifetime." (Wilber, 1963, p. 446).

Table 1. Expectations of life at birth and migration levels by region of residence and region of birth: United States female population, 1968.

A. Expectations of life at birth: ${}_ie_j(0)$

REGION OF BIRTH	REGION OF RESIDENCE				TOTAL
	1	2	3	4	
1. Northeast	54.13	5.08	10.11	5.25	74.56
2. North Central	3.76	52.14	10.48	8.05	74.44
3. South	5.06	7.88	54.53	6.93	74.40
4. West	3.90	7.94	11.32	52.41	75.57

B. Migration Levels: ${}_i\theta_j$

REGION OF BIRTH	REGION OF RESIDENCE				TOTAL
	1	2	3	4	
1. Northeast	0.7260	0.0681	0.1356	0.0704	1.00
2. North Central	0.0506	0.7005	0.1408	0.1081	1.00
3. South	0.0680	0.1060	0.7328	0.0931	1.00
4. West	0.0516	0.1051	0.1497	0.6936	1.00

Table 2. Net reproduction rates and allocations by region of residence and region of birth: United States female population, 1968.

A. Net reproduction rates: ${}_iNRR_j$

REGION OF BIRTH	REGION OF RESIDENCE				TOTAL
	1	2	3	4	
1. Northeast	0.8442	0.0727	0.1251	0.0647	1.1066
2. North Central	0.0521	0.8398	0.1312	0.1027	1.1258
3. South	0.0729	0.1167	0.8567	0.0893	1.1355
4. West	0.0533	0.1125	0.1472	0.7930	1.1061

B. Net reproduction allocations: ${}_i\rho_j$

REGION OF BIRTH	REGION OF RESIDENCE				TOTAL
	1	2	3	4	
1. Northeast	0.7628	0.0657	0.1131	0.0584	1.00
2. North Central	0.0463	0.7459	0.1166	0.0912	1.00
3. South	0.0642	0.1027	0.7545	0.0786	1.00
4. West	0.0482	0.1018	0.1331	0.7170	1.00

The application of Wilber's formula for calculating migration expectancies for individuals just born produces the direct analog of the conventional formula for the net reproduction rate. His Eq. 1, with x set equal to zero, may be expressed as

$$\sum_{x=0}^{z} L(x)M(x) \quad , \tag{1}$$

where L(x) is the stationary life table population aged x to x + 5 years at last birthday, z is the starting age of the last interval of life, and M(x) is the annual rate of migration among individuals in that age group. The corresponding formula for the net reproduction rate is

$$NRR = \sum_{x=0}^{z} L(x)F(x) \quad , \tag{2}$$

where F(x) is the age-specific fertility rate. The similarity between (1) and (2) suggests the designation of (1) as the net migraproduction rate, a quantity we shall denote by NMR. Thus NRR denotes the average number of babies per person, and NMR denotes the average number of moves per person, both taken over that person's entire lifetime. Observe that both measures depict the average number of occurrences of a recurrent event over an individual's lifetime.

A decade after Wilber's article, Long (1973) reported comparable migration expectancies, which despite a minor modification in the computational procedure produced similar results. Using 1966-71 data, he obtained, for example, a virtually identical value for a one-year-old's migration expectancy to a different house:

> "...at age one...a person...could expect 12.93 years with moves,..." (Long, 1973, p. 38).

Like Wilber before him, Long assumed that: (1) during the time interval over which migration was measured, each mover moved only once, and (2) persons reporting the same address at both the beginning and end of the interval did not move during the interval. Because of the first assumption, Long quite rightly observes that the calculated migration expectancy "does not quite represent the number of expected moves during an individual's remaining lifetime but instead represents the expected years with moves," (Long, 1973, p.38). Wilber recognized this problem but nevertheless continued to interpret his migration expectancies as expected number of moves, noting only that these probably understated the true number of moves. We shall do the same in this paper since such an interpretation more clearly reveals the correspondence between the net migraproduction and net reproduction rates.

3. The Spatial Net Migraproduction Rate

Earlier in this paper we proposed a spatial migration expectancy based on duration times, specifically, the expected number of years lived in region j by individuals born in region i. The correspondence between the net migraproduction and net reproduction rates suggests an alternative definition of spatial migration expectancy--one reflecting a view of migration as a recurrent event. Just as NRR was apportioned among the constituent regions of a multiregional system, so too can NMR be similarly disaggregated by place of birth and residence. Thus the formula for the spatial net reproduction rate:

$$ {}_iNRR_j = \sum_{x=0}^{z} {}_iL_j(x)F_j(x) \quad , \tag{3} $$

suggests the following definition for the spatial net migraproduction rate:

$$ {}_{i}NMR_{j} = \sum_{x=0}^{z} {}_{i}L_{j}(x)M_{j}(x) \quad , \tag{4} $$

where ${}_{i}L_{j}(x)$ denotes the stationary life table population of region j aged x to x + 5 years at last birthday and born in region i, and $M_{j}(x)$ is the age-specific out-migration rate in region j.

The spatial net migraproduction rate ${}_{i}NMR_{j}$ describes the average lifetime number of moves made out of region j by an individual born in region i. The summation of ${}_{i}NMR_{j}$ over all regions of destination ($j \neq i$) gives ${}_{i}NMR$, the net migraproduction rate of individuals born in region i, i.e., the average number of moves an i-born person is expected to make during his (or her) lifetime. Table 3 presents the matrix of net migraproduction rates for the four-region numerical illustration used earlier in Tables 1 and 2. Also included are the net migraproduction allocations

$$ {}_{i}\gamma_{j} = {}_{i}NMR_{j} / {}_{i}NMR \quad . \tag{5} $$

As with ${}_{i}\theta_{j}$ and ${}_{i}\rho_{j}$, high values of ${}_{i}\gamma_{j}$ ($i \neq j$) imply high levels of internal migration and vice versa.

4. Gross Rates and Consolidated Expectancies

Analogies are useful to the degree that they offer insights and suggest directions for the further extension of a new idea. Such appears to be the case here. We offer two possible extensions by way of illustration.

Table 3. Net migraproduction rates and allocations by region of residence and region of birth: United States female population, 1968.

A. Net migraproduction rates: ${}_iNMR_j$

REGION OF BIRTH	REGION OF RESIDENCE				TOTAL
	1	2	3	4	
1. Northeast	0.4178	0.0364	0.0520	0.0326	0.5387
2. North Central	0.0233	0.4665	0.0547	0.0510	0.5956
3. South	0.0320	0.0578	0.4116	0.0447	0.5460
4. West	0.0242	0.0575	0.0613	0.4649	0.6078

B. Net migraproduction allocations: ${}_i\gamma_j$

REGION OF BIRTH	REGION OF RESIDENCE				TOTAL
	1	2	3	4	
1. Northeast	0.7756	0.0675	0.0965	0.0604	1.00
2. North Central	0.0392	0.7833	0.0919	0.0857	1.00
3. South	0.0589	0.1058	0.7538	0.0818	1.00
4. West	0.0398	0.0946	0.1009	0.7648	1.00

4.1 The Gross Migraproduction Rate

Associated with the notion of the net reproduction rate (NRR) is the notion of the gross reproduction rate

$$GRR = 5 \sum_{x=0}^{z} F(x) \quad .$$

Does the notion of a gross migraproduction rate

$$GMR = 5 \sum_{x=0}^{z} M(x)$$

have a similarly useful interpretation?

The answer would seem to be yes. The GMR of a region measures the intensity of migration between it and another region at a particular point in time. The measure, therefore, has basically a cross-sectional character, in contrast to the NMR which measures the intensity of migration over a lifetime. Table 4 sets out the GMRs for our four-region numerical example. Note that our allocation index $\mathcal{E}$ excludes the diagonal in its denominator, i.e.,

$$_i\mathcal{E}_j = {}_iGMR_j / \sum_{j \neq i} {}_iGMR_j \quad . \qquad (6)$$

4.2 Consolidated Expectancies

The dominant characteristic root (or eigenvalue) of the matrix of spatial net reproduction rates may be interpreted as the net reproduction rate of the multiregional population as a whole; i.e., it is the consolidated net reproduction rate (Rogers and Willekens, 1975). Does it make sense to accord an analogous interpretation to the dominant characteristic root of the matrix of spatial net migraproduction

Table 4. Gross migraproduction rates and allocations by region of residence and region of birth: United States female population, 1968.

A. Gross migraproduction rates: ${}_iGMR_j$

REGION OF BIRTH	REGION OF RESIDENCE				TOTAL
	1	2	3	4	
1. Northeast	--	0.1258	0.3253	0.1377	0.5889
2. North Central	0.0978	--	0.3296	0.2526	0.6801
3. South	0.1462	0.2296	--	0.1853	0.5611
4. West	0.1005	0.2374	0.3186	--	0.6564

B. Gross migraproduction allocations: ${}_i\mathcal{E}_j$

REGION OF BIRTH	REGION OF RESIDENCE				TOTAL
	1	2	3	4	
1. Northeast	--	0.2137	0.5524	0.2339	1.00
2. North Central	0.1438	--	0.4847	0.3715	1.00
3. South	0.2605	0.4092	--	0.3303	1.00
4. West	0.1531	0.3616	0.4853	--	1.00

rates? Once again the answer apparently is yes. The dominant characteristic root of the 4 by 4 matrix of net migraproduction rates in Table 3 is 0.5785. The corresponding figure computed using the same data in consolidated form is 0.5721.

5. Conclusion

An individual's expectation of life at birth is the most commonly used indicator of a population's level of mortality. Because death is an event that occurs only once to an individual, the usual indicator of mortality is quite logically a measure of duration, i.e., the expected duration of an individual's life. Birth, however, is a potentially recurrent event. Consequently, fertility levels typically are measured in terms of the levels of reproduction that they imply. Thus the usual indicator of fertility is a count of the number of births to be experienced on the average by an individual member of the population.

Migration is potentially a recurrent event. Hence, like fertility, its level can be usefully measured in terms of a count of events, i.e., the expected number of moves per capita experienced by a particular population. However, migration levels also can be expressed in terms of expected durations, and the fraction of an individual's lifetime that is lived at a particular location is therefore an alternative indicator of geographical mobility. Both measures provide valuable insights into a population's migration behavior, and either may be used to classify different schedules of geographical mobility. For example, returning to Tables 1 and 3, we observe that the following two statements both describe, in their own way, the migration level from the West to the South regions:

1. A baby girl born in the West region and exposed over her lifetime to the multiregional schedules of migration and mortality that prevailed among U.S. women in 1968 could expect to experience 0.10 of her lifetime moves out of the South region.

2. A baby girl born in the West region and exposed over her lifetime to the multiregional schedules of migration and mortality that prevailed among U.S. women in 1968 could expect to live 0.15 of her lifetime in the South region.

Note that in both statements not only are the effects of mortality fused with those of migration, but also the effects of the mortality and migration schedules of the West region are confounded with those of all other regions via return migration. Consequently, the gross migraproduction rate often may prove to be a more useful measure than the net rate in that it is a "purer" indicator of migration, in the same sense as the gross reproduction rate is a purer measure of fertility than is the net reproduction rate. However, the gross rate measures the intensity of migration at a given moment and not over a lifetime. Hence, in instances where return migration is an important factor, the gross rate and the net rate may give differing indications of geographical mobility. For example, in Table 4 the allocation of the gross rate from the Northeast region to the South region is larger than the allocation to the same destination of the West region's gross rate (${}_1\mathcal{E}_3 = 0.5525 > {}_4\mathcal{E}_3 = 0.4853$). Yet the opposite is true of the corresponding allocations of the net rate in Table 3 (${}_1\gamma_3 = 0.0965 < {}_4\gamma_3 = 0.1008$). The cause of this reversal is the significantly higher return migration to the West region (${}_3\mathcal{E}_4 = 0.3302 > {}_3\mathcal{E}_1 = 0.2606$). Thus, because of the influence of return migration, the lifetime level of geographical mobility to the South region of a baby girl born in the Northeast region is lower, on 1968 rates of migration and mortality, than the corresponding mobility to the same destination of a baby girl born in the West region. The 1968 intensity of geographical mobility to the South region, however, was higher from the Northeast region than from the West region.

Acknowledgement

The author gratefully acknowledges the computational assistance of Frans Willekens.

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